CHAPTER TEN

RESOURCE ALLOCATION

Among the frequent operational problems in health care are resource allocation, service mix, scheduling and assignment. Linear Programming (LP) is an excellent tool to apply to those problems. In practice, software for nurse scheduling and operating room scheduling, empowered by linear programming and its extensions such as integer programming, provides optimal resource allocation and scheduling. In this chapter, we will describe both linear and integer programming applications in health care.

Linear Programming

Linear programming is a powerful tool that can incorporate many decision variables into a single model to attain an optimal solution. For example, a nurse scheduling problem in a medical center would involve many decision variables: various shift assignments and patterns, rotations, off days, weekend day designations, vacation requests, and holidays—all of which have to be considered simultaneously. When the requirements set up for health care management problems are translated into what is called constraints, it is possible for there to be so many that no solution to the problem appears to be feasible. However, health care managers can then reassess the requirements and relax some to seek possible solutions. To do that, one has to understand the nature of linear programming, and its structure. One must be able to observe simple problems (with few decision variables) graphically, and be able to conceptualize problems with many decision variables and constraints.

The structure of linear programming includes decision variables; an objective function; constraints; and the parameters that describe the available alternatives or resources.

The **decision variables** represent the levels of activity for an operation (for example, number of inpatient hospitalizations, number of outpatient visits); their values are determined by the solution of the problem. The variables are shown with symbols x_1, x_2, x_3, and so on in a linear equation. Decision variables cannot have negative values.

The **objective function** describes the goals the health care manager would like to attain (creating a reasonable margin for the survival or the financial health of the health care organization). Such a goal might be maximization of revenues or margins, or minimization of costs. The **objective function** is a linear mathematical statement of these goals (revenue, profit, costs) described in terms of decision variables (per unit of output or input). That is, the objective function is expressed as a linear combination of decision variables that will optimize the outcome (revenue, profit, costs) for the health care organization.

Constraints are the set linear equations that describe the limitations restricting the available alternatives and or resources. Especially in health care, scarce resources impede the management of facilities and/or the development of new health care services. The constraints to which the objective is subject arise from the health care organizations' operating environment. By factoring in constraints, a health care manager can see whether offering a new health care service would be feasible at all.

Parameters are the numerical values (values of available resources) that describe the fixed resources. Linear programming models are solved given the parameter values. This means that health care managers can emulate situations with "what if" questions by changing the values of the parameters in order to find alternative solutions. General structure of the linear programming model is as follows:

$$\text{Maximize (or minimize) } Z = c_1x_1 + c_2x_2 + c_3x_3 + \cdots + c_nx_n \qquad [10.1]$$

subject to

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n \ (\leq, =, \geq)\ b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n \ (\leq, =, \geq)\ b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n \ (\leq, =, \geq)\ b_3$$

$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n \ (\leq, =, \geq)\ b_m$$

$$x_i \geq 0 \qquad [10.2]$$

where

Z = objective function
x_i = decision variables
b_j = available resource for jth constraint
c_i = objective function coefficients
a_{ij} = coefficient for ith decision variable on jth constraint.

Maximization Models

To illustrate these concepts in an example and build a linear programming model for it, consider the following maximization example.

Example 10.1

An insurance company desires to enter the health care market and offer its potential customers both a staff model health maintenance organization (HMO) and commercial indemnity insurance. The company is deciding how to allocate its marketing efforts between those options to maximize its profits. The analysts have estimated that the company will realize a profit of $1,200 per enrollee from the HMO, and $600 per enrollee from commercial plans. Furthermore, for the coming year the company is forced to rely on its present resources in terms of sales force. The administrative support of the HMO will take two hundred hours, and the commercial administration will take, on average, four hundred hours; currently, the company can allocate 1.6 million hours to sales. To break even, the HMO requires that the contribution margins (contribution margin is sales revenue less variable costs; it is the amount available to pay for fixed costs and then provide any profit after variable costs have been paid) for enrollees must exceed $1.5 million. The estimated contribution margins are $500 and $300, for HMO and for commercial insurances enrollees, respectively. With a limited number of physicians participating in the staff model HMO at the present time, the HMO can handle at most 5,000 enrollees.

Solution: To formulate the model for this problem, first we must identify the decision variables. In this case the two options, HMO and indemnity insurance, are the decision variables. The number of enrollees required for profitable operations is determined by the level of activity in each of those variables. Let us assign a symbol of x_1 to indicate the potential number of HMO enrollees; similarly let x_2 represent the enrollees in the indemnity plan.

The next step is to express the objective function in a linear fashion to represent the maximum profits for each of those decision variables. Recall that the company was expecting, respectively, $1,200 and $600 profit from each HMO and each indemnity enrollee. The objective function is the summation of these expectations and can be formulated as:

$$\text{Maximize } Z(\text{profit}) = 1{,}200x_1 + 600x_2.$$

Once the objective function is determined, the constraints it is subject to must be developed. It is indicated that the insurance company will use its existing resources to develop marketing campaigns for those new products, but the resources are limited by the parameters. For example, the available administrative support is limited to 1.6 million hours of staff time. We have to convert that information into a constraint; let us call it the administrative support constraint. To express the constraint as x_1, x_2, we must note the rate at which each product would consume the resource. In the problem those rates are given as two hundred hours for the HMO and four hundred hours for the indemnity plan, respectively.

The formulation of the administrative support constraint is then:

$$200x_1 + 400x_2 \leq 1{,}600{,}000 \text{ (administrative support constraint).}$$

This constraint indicates that the linear combination of enrollees from both plans can be administratively supported up to 1,600,000 hours from existing resources.

The second constraint in the problem assures a minimum of \$1,500,000 as the contribution margin, with \$500 from each HMO enrollee and \$300 from each indemnity enrollee, and is written as:

$$500x_1 + 300x_2 \geq 1{,}500{,}000 \text{ (contribution margin constraint).}$$

It should be noted that this constraint has the sign greater than equal at the right end side of the equation indicating that expectation for the contribution margin is at minimum that amount (\$1.5 million).

The final constraint of this problem is how many enrollees the company can handle at the start with the given resources. There is no restriction on indemnity enrollees, but for the HMO only 5,000 enrollees are permitted. Hence, this last equation can be expressed as:

$$1x_1 + 0x_2 \leq 5{,}000 \text{ (enrollees constraint).}$$

Since none of the decision variables can have a negative value, we must enforce a non-negativity constraint on the variables as:

$$x_1, x_2 \geq 0.$$

Summarizing the development so far, we have a linear programming formulation of this problem:

$$\text{Maximize } Z(\text{profit}) = 1{,}200x_1 + 600x_2$$

Subject to:

$$200x_1 + 400x_2 \leq 1{,}600{,}000 \quad \text{(administrative support constraint)}$$
$$500x_1 + 300x_2 \geq 1{,}500{,}000 \quad \text{(contribution margin constraint)}$$
$$1x_1 + 0x_2 \leq 5{,}000 \quad \text{(enrollees constraint)}$$
$$x_1, x_2 \geq 0$$

The next step is to plot the constraints and identify an area that satisfies all the constraints, called the **feasible solution space.** Then one plots the objective function to determine the **optimal solution** in the feasible solution space. The following steps describe the graphical approach and the solution to this problem.

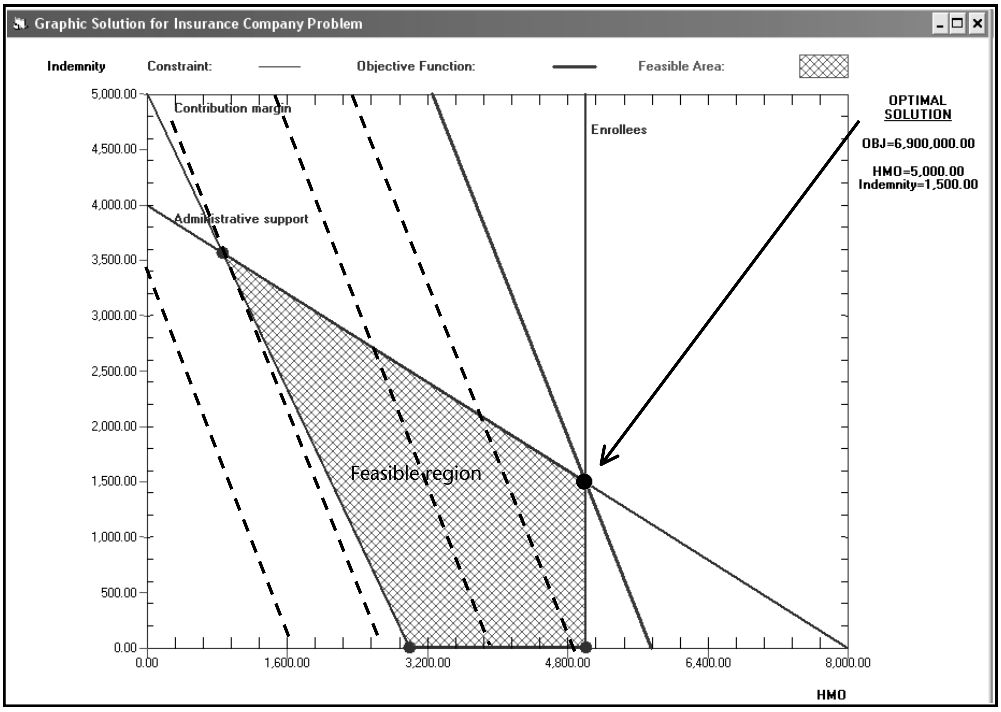
Step 1: plot the identified constraints: determine where the line intersects each axis. Mark those intersections and connect them. Close attention must be placed to whether a constraint is a less-than or greater-than constraint. For instance, for the administrative support constraint, the intercepts are $x_1 = 8{,}000$ (determined by setting $x_2 = 0$ and solving for x_1: $1{,}600{,}000 \div 200 = 8{,}000$); and $x_2 = 4{,}000$ (determined by setting $x_1 = 0$ and solving for x_2: $1{,}600{,}000 \div 400 = 4{,}000$). Because it is a ≤ constraint, the area between the origin and this line is the feasible solution space.
Step 2: continue plotting all constraints to identify the total feasible solution space.
Step 3: plot the objective function and observe where it has the highest value (maximization) while still touching (tangent to) the feasible solution space. This is the location of the optimal solution. ■

The graphic presentation of this problem is shown in Figure 10.1. The first constraint, administrative support, is a ≤ type constraint, which means that the feasible solution must occur below the line and toward to the origin point (0.0). On the other hand, the second constraint, contribution margin, is the ≥ type, which means that the feasible area must be above the line and away from the origin. Finally, the third constraint, enrollees, represents restriction in only one variable, and is a ≤ type constraint, so once again the feasible region must occur below the line and towards the origin.

The dashed parallel lines show the iso-profit (objective function) values. The goal is to maximize the profit by choosing the iso-profit line that has the highest value. In maximization problems, the iso-objective function line that is tangent to the feasible solution space at the farthest point yields the greatest value for the objective function, and provides the optimal solution.

The solution to this problem is also displayed in Figure 10.1, where resources are allocated to each program. That is, the insurance company should have 5,000 HMO and 1,500 indemnity enrollees to maximize its profit at $6,300,000 without violating any of the imposed constraints (limitations). It is noteworthy here that the solution to this problem occurs at the intersection of administrative support and

FIGURE 10.1. GRAPHIC SOLUTION FOR INSURANCE COMPANY PROBLEM.



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enrollee constraints (first and third constraints). In linear programming terminology, a constraint that forms the optimal corner point of the feasible solution space is called the binding constraint. Here those two constraints are the binding constraints: any change in their right-hand side values, b_j, would immediately affect the objective function value and the solution. On the other hand, the nonbinding constraints, in this case the contribution margin constraint, do not affect the final solution unless a dramatic change occurs in the parameters.

Although the graphed solution to linear programming problems is illustrative and easy to understand, when there are more than two decision variables in the model, graphical solutions are no longer practical and linear algebraic methods are required. A method that is instrumental for obtaining optimal solutions to linear programming problems is the **simplex method.** This methodology is embedded in computer software programs that solve linear programming problems. WinQSB has a linear programming module, and we will follow the computer-based solution using this software.

FIGURE 10.2. WINQSB DATA ENTRY AND SOLUTION TO THE INSURANCE PROBLEM.

Insurance Company Problem — Data entry

Maximize : Direction

Variable -->	HMO	Indemnity	Direction	R. H. S.
Maximize	1200	600		
Administrative support	200	400	<=	1600000
Contribution margin	500	300	>=	1500000
Enrollees	1	0	<=	5000
LowerBound	0	0		
UpperBound	M	M		
VariableType	Continuous	Continuous		

Solution

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	HMO	5000.00	1200.00	6000000.00	0	basic	300.00	M
2	Indemnity	1500.00	600.00	900000.00	0	basic	0	2400.00
	Objective	Function	(Max.) =	6900000.00				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	Administrative support	1600000.00	<=	1600000.00	0	1.50	1000000.00	M
2	Contribution margin	2950000.00	>=	1500000.00	1450000.00	0	-M	2950000.00
3	Enrollees	5000.00	<=	5000.00	0	900.00	857.14	8000.00

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Figure 10.2 depicts the data entry (upper section) and the solution (lower section) to this problem. Two decision variables, HMO and indemnity, are identified, and the first row of the data shows the objective function of the problem where maximization is sought. The coefficients (c_i) of each decision variable (1,200 and 600) in the objective function are shown in the first row. The following three rows depict the constraints with their directions and right-hand side (R.H.S) values (b_j). The coefficients for each variable on a given constraint, a_{ij}, are also shown (200 and 400 for the administrative support constraint).

The lower part of Figure 10.2 provides the combined summary results of the WinQSB solution, which require explanation to interpret them and further analyze the problem. The solution value for each decision variable, the unit profit values and the total contribution to the objective function are shown in the labeled columns. That is, with 5,000 enrollees at $1,200 per enrollee for the HMO, the total contribution to the objective function from this variable is $6,000,000. The remaining $900,000 is contributed by the indemnity product with 1,500 enrollees, each bringing $600 profit. Thus, the total profit amounts to $6,900,000 with this solution. A "0" (zero) in the "Reduced Costs" column indicates that no further improvement is possible for the objective function from this variable unless the right-hand side (resources) improves. The word "basic" in the "Basic Status" column indicates that the particular decision variable is in the final solution and thus contributes to the objective function. There are instances in which not all decision variables contribute to the final solution. The "Allowable Minimum c_i" and "Allowable Maximum c_i" columns show the range of each decision variable for the objective function. In this example, profit per enrollee cannot be lower than $300 for the HMO, but can be infinitely high (M stands for a very large number). Similarly, for the indemnity product, profit can go as low as 0, but cannot be higher than $2,400 per enrollee.

The last part of the solution section in Figure 10.2 shows the constraints, their allowable values and their effects on the objective function. Recall that the intersection of the first and third constraints (administrative support, and enrollees) defined the optimal solution to this problem. These are binding constraints or tight constraints, which means that they cannot be moved to the left or right (in the graph) without affecting the solution. Notice that the values in the columns "Left-Hand Side" and "Right-Hand Side" for those two constraints are equal. However, the LHS and RHS values are different for the nonbinding constraint (contribution margin). These observations lead to a discussion of slack, surplus, shadow prices and range of feasibility in linear programming. Let us define each of those concepts.

Slack—When the optimal values of decision variables are substituted into a ≤ constraint, and the resulting value is less than the right-hand side value.

Surplus—When the optimal values of decision variables are substituted into a ≥ constraint, and the resulting values exceed the right-hand side value.

Shadow Prices—How much a one-unit increase in the right-hand side of a constraint would increase the value of the objective function.

Range of Feasibility—The range of values for the right-hand side of a constraint over which the shadow price remains the same.

In Figure 10.2, the lower set of columns in the solution section, for constraints depicts the values of those concepts just defined above. The second, and only nonbinding, constraint: "contribution margin," has 145,000 under the "Slack or Surplus" column. Since this constraint is a ≥ type constraint, that is the amount of surplus; one could increase the right-hand side of the equation by this amount (to 295,000) without violating the existing solution. "Shadow Price" of enrollees appears as 900, indicating that every additional enrollee (beyond 5,000) can improve profits by $900. If the number of physicians to handle more than 5,000 HMO enrollees were not subject to restrictions, the insurance company could enroll up to 8,000 enrollees and generate additional 2,700,000 (3,000 * 900) profit.

On the other hand, the company cannot afford to enroll fewer than 857. The shadow price of administrative support is interpreted similarly, although the unit contribution to profit (objective function) is dramatically smaller at $1.50. Having said that, one should note that if the human resources are available (in hours), the contribution to profit is infinite. One then must make a cost-benefit analysis as to whether it is worthwhile to expand one extra hour of human resource to generate $1.50 in additional profit.

Minimization Models

When the measures in the objective function are costs, obviously health care managers seek to minimize those costs. Model set-up follows the same steps, with one exception: in cost minimization problems, the constraints are generally the ≥ type. Thus, in the graphic solution, the feasible area is defined from infinity towards origin.

EXAMPLE 10.2

$$\text{Minimize } Z = 60x_1 + 30x_2$$

Subject to:

$$20x_1 + 40x_2 \geq 160 \text{ C1 (constraint 1)}$$
$$40x_1 + 30x_2 \geq 240 \text{ C2 (constraint 2)}$$
$$x_1, x_2 \geq 0.$$

The graphical solution to this minimization problem is shown in Figure 10.3.

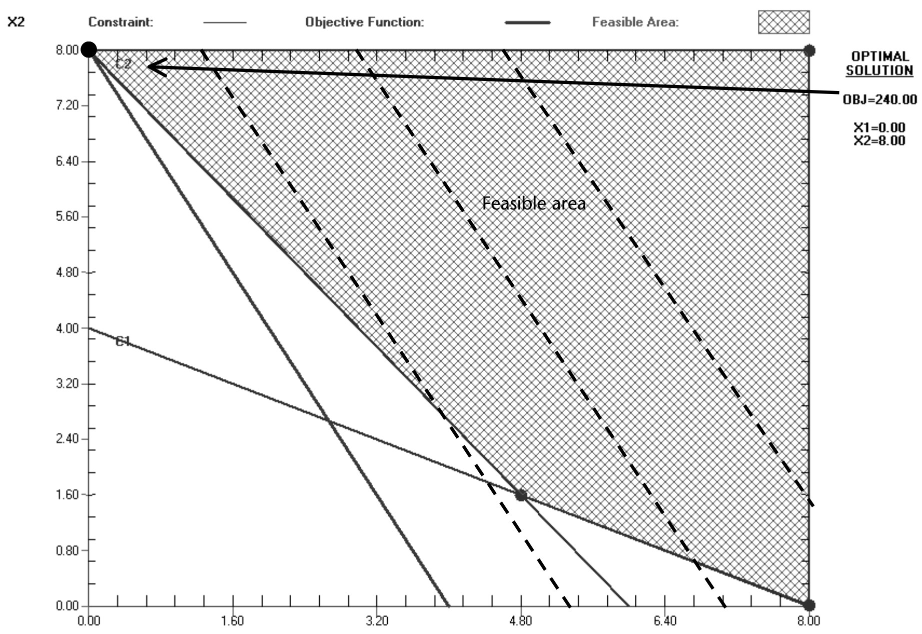
Here, the optimal solution occurs at $x_2 = 8$ and $x_1 = 0$. The objective function, with its slope, just clears the feasible region (tangent) at this point. It should be noted that the objective function (the iso-cost lines) is coming down from higher values (cost) to this value, which is the minimum for this problem.

Finally, Figure 10.4 depicts the WinQSB solution to the minimization example.

Integer Linear Programming

In linear programming one of the assumptions is that decision variables are continuous. Therefore solutions can yield fractional values such as 4.3 patients, or 7.6 nurses. Such solutions are especially impractical, however, when linear programming is used for scheduling the clinical staff. Rounding off these values may generate infeasible or less optimal solutions. Integer programming is an extension

FIGURE 10.3. GRAPHIC SOLUTION FOR MINIMIZATION EXAMPLE.



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of linear programming that eliminates the problem by enforcing integer decision variable outcomes.

Health care facilities usually provide service around the clock seven days a week, so scheduling staff is a significant operational task for clinic managers. Many factors must be included in the model so that an equitable schedule can be produced. A typical full-time employee works five days a week with two days off. Although the off days can be either consecutive or spread during the week according to resource availability, clinical staff generally prefer two consecutive days off, for rotating weekends. Each clinical unit has minimum staffing requirements (core staff) for each shift. The aim of management is to meet the core coverage of each day and shift while satisfying the schedule of 5 work days and 2 consecutive days off for each staff member.

Let us illustrate a simple version of staff scheduling. In linear integer programming, scheduling can be thought of as cycles (tours) of assignments. Since the most critical element of the scheduling is deciding on the off days, the decision variables can be conceptualized as the two off days that a staff member is assigned in a scheduling cycle. There are seven possible pairs of consecutive off days available:

FIGURE 10.4. SENSITIVITY ANALYSIS.

Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price
Administrative support	1600000.00	<=	1600000.00	0	1.50
Contribution margin	2950000.00	>=	1500000.00	1450000.00	0
Enrollees	5000.00	<=	5000.00	0	900.00
Objective	Function	(Max.) =	6900000.00		

Constraint	Direction	Shadow Price	Right Hand Side	Allowable Min. RHS	Allowable Max. RHS
Administrative support	<=	1.50	1600000.00	1000000.00	M
Contribution margin	>=	0	1500000.00	-M	2950000.00
Enrollees	<=	900.00	5000.00	857.14	8000.00

Decision Variable	Solution Value	Reduced Cost	Unit Cost or Profit C(j)	Allowable Min. C(j)	Allowable Max. C(j)
HMO	5000.00	0	1200.00	300.00	M
Indemnity	1500.00	0	600.00	0	2400.00

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Saturday-Sunday, Sunday-Monday, Monday-Tuesday, Tuesday-Wednesday, Wednesday-Thursday, Thursday-Friday, and Friday-Saturday. If we can make the assignments to guarantee these off days to clinical staff while meeting the unit staffing level requirements for each day, we will have produced a satisfactory schedule.

A formal formulation of integer linear programming for staff assignments is as follows (adapted from Fitzsimmons & Fitzsimmons, 2004; p. 255):

$$\text{Minimize } Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$$

Subject to:

$$\begin{aligned}
x_1 + x_2 + x_3 + x_4 + x_5 &\geq b_1 \text{ Saturday constraint} \\
x_2 + x_3 + x_4 + x_5 + x_6 &\geq b_2 \text{ Sunday constraint} \\
x_3 + x_4 + x_5 + x_6 + x_7 &\geq b_3 \text{ Monday constraint} \\
x_1 + x_4 + x_5 + x_6 + x_7 &\geq b_4 \text{ Tuesday constraint} \\
x_1 + x_2 + x_5 + x_6 + x_7 &\geq b_5 \text{ Wednesday constraint} \\
x_1 + x_2 + x_3 + x_6 + x_7 &\geq b_6 \text{ Thursday constraint} \\
x_1 + x_2 + x_3 + x_4 + x_7 &\geq b_7 \text{ Friday constraint} \\
x_i &\geq 0 \text{ and integer}
\end{aligned}$$

where

Z = objective function

x_i = decision variables (x_1 = off on Saturday & Sunday, x_2 = off on Sunday & Monday, etc.)

b_j = minimum staff requirements for a day of the week (b_1 = required staff for Saturday).

To further illustrate staff scheduling, consider the following example:

Example 10.3

A nurse manager must schedule staff nurses in a rehab unit. Nurses work five days a week with two consecutive off days. The staff requirements of the nursing unit are seven nurses for each day of the week. The nurse manager wants an equitable schedule for all the staff while meeting the unit staff requirements each day.

Solution: Since this problem has more than two decision variables, a graphic solution is not possible. A computer solution using WinQSB will be provided. Figure 10.5 displays the data entry and the solution to this problem.

As the upper portion of the exhibit depicts, the seven decision variables are the pairs of off days, and the right-hand side (RHS) of each day constraint shows the staff requirement for that day. It is more challenging to interpret the results, shown in the lower portion of the exhibit, to develop the schedule.

The solution for each decision variable indicates how many cycles (tours) are needed to satisfy the daily staffing requirement for the unit while assuring a

TABLE 10.1. NURSE SCHEDULING WITH INTEGER PROGRAMMING.

Nurse ID	Sat	Sun	Mon	Tue	Wed	Thu	Friday
1	O	O	A	A	A	A	A
2	O	O	A	A	A	A	A
3	A	O	O	A	A	A	A
4	A	A	O	O	A	A	A
5	A	A	O	O	A	A	A
6	A	A	A	O	O	A	A
7	A	A	A	A	O	O	A
8	A	A	A	A	O	O	A
9	A	A	A	A	A	O	O
10	O	A	A	A	A	A	O
Required	7	7	7	7	7	7	7
Assigned	7	7	7	7	7	7	8
Excess	0	0	0	0	0	0	1

pair of days off to each staff nurse. Explicitly, $x_1 = 2$ indicates that the nurse manager should assign two nurses with Saturday-Sunday off; $x_2 = 1$ indicates that one nurse should be assigned to have Sunday-Monday off; $x_3 = 2$ indicates that two nurses should be assigned to have Monday-Tuesday off. With that information, a nurse manager can draft a schedule.

Table 10.1 shows the resulting schedule, with A for assignments and O for days off. The last few rows of the table show the requirements and assignments, the total number of "A"s in a given day, and any excess assignment for a day. To implement this schedule a total of 10 nurses are needed, which is the number given by the objective function value shown in the solution (Figure 10.5). ■

FIGURE 10.5. STAFF SCHEDULING WITH INTEGER LINEAR PROGRAMMING.

Variable -->	Sat-Sun	Sun-Mon	Mon-Tue	Tue-Wed	Wed-Thu	Thu-Fri	Fri-Sat	Direction	R. H. S.
Minimize	1	1	1	1	1	1	1		
Saturday	1	1	1	1	1			>=	7
Sunday		1	1	1	1	1		>=	7
Monday			1	1	1	1	1	>=	7
Tuesday	1			1	1	1	1	>=	7
Wednesday	1	1			1	1	1	>=	7
Thursday	1	1	1			1	1	>=	7
Friday	1	1	1	1			1	>=	7
LowerBound	0	0	0	0	0	0	0		
UpperBound	M	M	M	M	M	M	M		
VariableType	Integer	Integer	Integer	Integer	Integer	Integer	Integer		

Decision Variable	Solution Value	Unit Cost or Profit C(j)	Total Contribution	Reduced Cost	Basis Status
Sat-Sun	2.0000	1.0000	2.0000	0.3333	at bound
Sun-Mon	1.0000	1.0000	1.0000	0	basic
Mon-Tue	2.0000	1.0000	2.0000	0	basic
Tue-Wed	1.0000	1.0000	1.0000	0	basic
Wed-Thu	2.0000	1.0000	2.0000	0	basic
Thu-Fri	1.0000	1.0000	1.0000	0	basic
Fri-Sat	1.0000	1.0000	1.0000	0	basic
Objective	Function	(Min.) =	10.0000		

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Summary

Resource allocation can take the form of distribution of beds, products, staff and other resources in various health services. Linear programming and its extensions provide optimal solutions to allocation problems. In practice, these methods often are imbedded into scheduling software that is used by divisional or departmental managers.

Exercises

Exercise 10.1

Given the following linear programming formulation:

$$\text{Maximize } 1600x_1 + 3000x_2$$

Subject to:

$$40x_1 + 25x_2 \leq 80{,}000 \text{ (constraint 1)}$$
$$20x_1 + 30x_2 \leq 60{,}000 \text{ (constraint 2)}$$
$$x_1, x_2 \geq 0 \text{ (non-negativity constraints).}$$

a. Solve the problem graphically.
b. Solve the problem using computer-based linear programming software (WinQSB).
c. What is the total objective function value?
d. Do both variables contribute to the solution? Why?
e. Does any variable have a slack value? If so, what does it mean?

Exercise 10.2

The cost of providing public services at local hospital has been scrutinized by management. Although these services are used as marketing tools for the hospital, the cost and availability of scarce resources require their optimal allocation while minimizing costs. Two popular programs being assessed for this purpose are "Family Planning" (FP) and "Health-Drive-Screenings" (HDS); their costs to the hospital for each offering are $200 and $400, respectively. The health care manager in charge of operations found three common patterns of resource consumption for each of these services and the available resources, shown in Table EX 10.2.

TABLE EX 10.2.

Resource Type	FP	HDS	Available Resources per Month
Staff time	60	120	480 minutes
Materials	30	90	250 kits
Rent space		1	3 occasions

a. Formulate this as a linear programming problem.
b. Solve the problem graphically.
c. Solve the problem using computer-based linear programming software (WinQSB).
d. In a given month, how many FP and how many HDS should be offered?
e. With the proposed class offerings, how many kits will be left over (not distributed in the classes)?
f. What is the yearly cost of these two programs to the hospital?

Exercise 10.3

A practice would like to allocate their resources optimally between the orthopedic and rheumatology departments. The revenues per case generated by orthopedics and by rheumatology are $2,000 and $1,000, respectively. The average number of visits, utilization of radiology resources per case, and available resources are in Table EX 10.3.

TABLE EX 10.3.

	Orthopedics	Rheumatology	Available Resources
Visits	2	3	600 hours MD time
Radiology	4	1	800 procedures

a. Formulate this as a linear programming problem.
b. Solve the problem graphically.
c. Solve the problem using computer-based linear programming software (WinQSB).
d. For the optimal solution, what should be the percentages of allocation between the two departments?
e. How much total combined revenue can be generated with this solution?

Exercise 10.4

A hospital is evaluating the feasibility of offerings among three technologies, on the basis of what would make the most profit. These new technologies are:

1) Closed-chest cardiac bypass surgery with "da Vinci Surgical Robot."
2) Gamma knife.
3) Positron emission tomography (PET) Scanner.

Table EX 10.4 gives the information on profit, the amount of common resources used by each of the three technologies per case, and their available resources per month:

TABLE EX 10.4.

	da Vinci	Gamma K.	PET	Available Resources
Profit $	2,000	3,500	2,000	
Total staff time	15	12	1.5	2,000 hours
Maintenance	25	25	22	1,500 minutes
Computer resources	20	25	10	3,000 minutes

a. Formulate this as a linear programming problem.
b. Solve the problem using computer-based linear programming software (WinQSB).
c. Based on optimal solution, which product(s) should be offered, and how many procedures can be offered in a month?
d. What is the expected contribution of new technology to the hospital's monthly profits?

Exercise 10.5

A community hospital is planning to expand its services to three new service lines in the medical diagnostic categories (MDCs) and their corresponding diagnostic related groupings (DRGs) shown in Table 10.5.1

TABLE EX 10.5.1.

MDC	DRGs	Description
2	36–47	Diseases and disorders of the eye
19	424–433	Mental diseases and disorders
21	439–455	Injury, poisoning, and toxic effects of drugs

Five common resources must to be allocated among these three new service lines according to which will bring the most revenue (using overall average DRG payments in a given MDC category). The resources are beds (measured as patient days), nursing staff, radiology, laboratory, and operating room (hint: constraints). The health care manager in charge of this expansion project obtained the average consumption patterns of these resources for each MDC from other peer institutions, and estimated the resources that can be made available (per year) for the new service lines in Table EX 10.5.2:

TABLE EX 10.5.2.

Resource Category	MDC-2	MDC-19	MDC-21	Available Resources
Length of stay (LOS)	3.3	6.1	4.4	19,710
Nursing hours	3	5	4.5	16,200
Radiology procedures	0.5	1.0		3,000
Laboratory procedures	1	1.5	3	6,000
Operating room	2		4	1,040

Average revenues from MDC-2, MDC-19, and MDC-21 are $8,885, $10,143, and $12,711, respectively.

a. Formulate this as a linear programming problem.
b. Solve the problem using computer based linear programming software (WinQSB).
c. To get the most revenue, which service(s) should be offered?
d. What is the optimal volume(s)?
e. What is the total expected revenue from the new services?

f. Which resources should be expanded?
g. How much additional revenue can be expected if resources are selected in "f" for expansion without violating the current solution?

Exercise 10.6

A regional laboratory that performs nontraditional tests is planning to offer new diagnostic tests for regional hospitals. Current analyzers and staff are capable of performing these tests. The laboratory manager assessed the required staff and analyzer times, as well as the chemical materials required for a bundle of 50 vials for each type of test listed in Table EX 10.6.

TABLE EX 10.6.

Test Type ➔	I	II	III	IV	V	Available Resources
Profit ($)	8	10	8	7	10	
Staff (minutes)	15	15	15	20	25	3,400
Auto analyzer equipment (minutes)	20	40	40	60	45	6,000
Materials	12	15	16	14	14	2,700

a. Formulate this as a linear programming problem.
b. Solve the problem using computer-based linear programming software (WinQSB).
c. For the optimal solution, in terms of profit, which test(s) should be offered?
d. What is the optimal volume(s)?
e. What is the total expected profit from the new tests?
f. Which resources should be expanded?

How much additional revenue can be expected if the resources are selected in "f" for expansion without violating the current solution?